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Engineering Notes

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Semiglobal Trajectory Tracking Control Law for Nonlinear Nonminimum Phase Three-Degree-of-Freedom Flight Vehicle

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Introduction

NONMINIMUM phase systems are encountered in many engineering applications, such as flexible robot control [1,2], ship control [3], aircraft control [4–8], and missile control [9–11]. It is known that nonlinear, nonminimum phase systems provide fundamental obstacles to design of high-performance control systems. A number of control design methods have been proposed in the literature to handle nonminimum phase systems.

Gain scheduling is widely used in designing flight control systems [12,13]. The main advantage of the gain scheduling approach is that a wealth of linear design methods can be applied to the linearized models at each local flight condition. Output redefinition [14] can result in a new system model that is minimum phase. This method requires that a new output is selected that is "close" to the original output in some physical sense. Dynamic sliding manifolds [15] are another approach to output tracking for nonlinear nonminimum phase systems. This method joins features of a conventional sliding mode controller and a conventional dynamic compensator. Applications of this method to flight control problems can be found in [16]. The resulting control laws are discontinuous and may cause chattering. The flat output approach used in [6] also involves output redefinition by finding new outputs (different from the desired outputs) such that the nonlinear system is feedback linearizable with respect to the new output. This approach allows the system states and controls to be written as a function of the flat outputs and their derivatives.

The output regulation method [17] is a general method that is applicable to nonlinear, nonminimum phase tracking problems provided that the desired commands are generated by an exosystem. This method is an extension to the problem of output tracking for linear time-invariant systems that was solved by Francis [18]. The noncausal stable inversion method [19] is an iterative method that determines a bounded control inverse for a nonlinear system. In the case of nonminimum phase systems, the inverse is noncausal, so that the complete history of the desired output must be known to find a bounded inverse. The noncausal stable inversion approach has been applied to flight control problems; see, for example, [20,21]. Maneuver regulation or path-following controllers have been studied for robotic systems [22,23] and for aerospace vehicles [24–26]. A general approach has been developed in [27,28] for a feedback linearizable nonlinear control system and in [24] for nonminimum phase systems.

In this paper, we present a new approach to design a semiglobal trajectory tracking controller for a 3-degree-of-freedom (3DOF) aircraft model such that the aircraft outputs track a desired output with internal stability. In this approach, we separate the aircraft dynamics into fast and slow subsystems using two timescale feedback control. First, a high-gain feedback linearizing controller is designed for the fast subsystem such that the tracking error is forced to zero relatively fast. Second, a feedback linearizing controller is designed for the slow subsystem to derive the tracking error to zero asymptotically. The second controller is designed under the assumption that the first output has converged to its desired value relatively fast. This approach has the advantage of achieving a lower order controller compared to the dynamic extension approach described in [6,29]. In fact, the dynamic extension approach will result in an eight-dimensional system compared to six in this approach.

Mathematical Model of Aircraft

The 3DOF helicopter with two propellers is shown in Fig. 1. The helicopter can rotate horizontally around point A with a yaw angle ψ. It can also pitch up and down around point A with a pitch angle θ. Also, two identical propellers are mounted on a bar that is suspended on beam M. The bar can rotate in the plane perpendicular to beam M with a roll angle ϕ.

We approximate the model of the 3DOF helicopter so that it has the same dynamics as that of the vertical takeoff and landing aircraft model [4,5]. The simplified equations of motion for the 3DOF helicopter can be written in this case as

$$\ddot{\theta} = u_s \cos\phi + \epsilon u_d \sin\phi - \Omega^2 \tag{1}$$

$$\ddot{\psi} = -u_s \sin\phi + \epsilon u_d \cos\phi \tag{2}$$

$$\ddot{\phi} = u_d \tag{3}$$

where

$$u_s = \frac{l_1}{I_{\theta\theta}}(F_1 + F_2) \qquad u_d = \frac{d}{I_{\phi\phi}}(F_1 - F_2)$$

$$\Omega^2 = \frac{g}{I_{\theta\theta}}(Ml_0 - M_c l_c + 2m_r l_1), \qquad \omega^2 = \frac{2m_r gh}{I_{\phi\phi}}, \qquad \epsilon = \frac{l_1 I_{\phi\phi}}{dI_{\theta\theta}}\kappa$$

where d,h,l_0,l_1, and l_c are lengths from the vehicle geometry, and M, M_c, and m_r are masses representing different components as shown in Fig. 1. The moment of inertia about pitch and roll are denoted by $I_{\theta\theta}$ and $I_{\phi\phi}$, respectively. Note that we have assumed the pitch and yaw moments of inertia are equal. The thrust forces developed by the two propellers are denoted by F_1 and F_2. The angle κ is assumed small and represents thrust misalignment.

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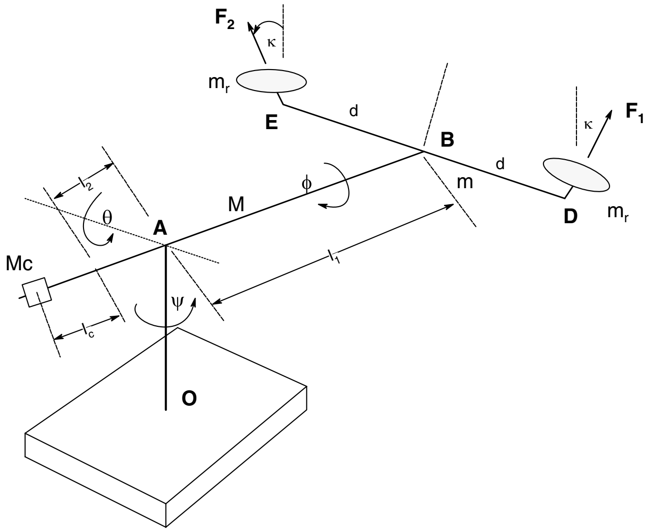

The 3DOF helicopter

Fig. 1 Model aircraft and notations.

Flight Control Design

Control Objectives

The control objective is to find a control law (u_s, u_d) such that the aircraft tracks a given path $(\psi_c(t), \theta_c(t))$ with closed-loop stability. The tracking problem for the aircraft can be stated mathematically as the following: Given sufficiently smooth functions $(\psi_c(t), \theta_c(t))$ in the inertial frame, find a bounded control law (u_s, u_d) such that the following holds:

1) $\psi(t) - \psi_c(t) \to 0$ as $t \to \infty$.
2) $\theta(t) - \theta_c(t) \to 0$ as $t \to \infty$.
3) Closed-loop stability.

Input/Output Linearization and Zero Dynamics

In this section, we first introduce a coordinate transformation to rewrite Eqs. (1) and (2) in terms of error coordinates. Second, we introduce a coordinate transformation for Eq. (3) so that the roll dynamics do not depend explicitly on the control. For this purpose, the six-dimensional state vector can be obtained using the following state coordinate transformation:

$$\begin{bmatrix} e_1 \\ e_2 \\ e_3 \\ e_4 \\ \eta_1 \\ \eta_2 \end{bmatrix} = \begin{bmatrix} \psi - \psi_c \\ \dot{\psi} - \dot{\psi}_c \\ \theta - \theta_c \\ \dot{\theta} - \dot{\theta}_c \\ \phi \\ \epsilon\dot{\phi} - (\dot{\psi} - \dot{\psi}_c)\cos\phi - (\dot{\theta} - \dot{\theta}_c)\sin\phi \end{bmatrix} \tag{4}$$

Note that the preceding coordinate transformation is well defined as long as $\epsilon \neq 0$.

Further, we introduce new inputs defined by the invertible feedback transformation

$$\begin{bmatrix} u_s \\ u_d \end{bmatrix} = \begin{bmatrix} (\upsilon_2 + \ddot{\theta}_c + \Omega^2)\cos\phi - (\upsilon_1 + \ddot{\psi}_c)\sin\phi \\ \frac{1}{\epsilon}[(\upsilon_2 + \ddot{\theta}_c + \Omega^2)\sin\phi + (\upsilon_1 + \ddot{\psi}_c)\cos\phi] \end{bmatrix} \tag{5}$$

or

$$\begin{bmatrix} \upsilon_1 \\ \upsilon_2 \end{bmatrix} = \begin{bmatrix} \epsilon u_d\cos\phi - u_s\sin\phi - \ddot{\psi}_c \\ \epsilon u_d\sin\phi + u_s\cos\phi - (\ddot{\theta}_c + \Omega^2) \end{bmatrix} \tag{6}$$

Using the state and control transformations (4–6), Eqs. (1–3) can be written in normal form [30] as

$$\dot{e}_1 = e_2 \tag{7a}$$

$$\dot{e}_2 = \upsilon_1 \tag{7b}$$

$$\dot{e}_3 = e_4 \tag{8a}$$

$$\dot{e}_4 = \upsilon_2 \tag{8b}$$

$$\dot{\eta}_1 = \frac{1}{\epsilon}[\eta_2 + e_2\cos\eta_1 + e_4\sin\eta_1] \tag{9a}$$

$$\begin{aligned} \dot{\eta}_2 = {} & \frac{1}{\epsilon}[\eta_2 + e_2\cos\eta_1 + e_4\sin\eta_1][e_2\sin\eta_1 - e_4\cos\eta_1] \\ & + \ddot{\psi}_c\cos\eta_1 + (\Omega^2 + \ddot{\theta}_c)\sin\eta_1 \end{aligned} \tag{9b}$$

Equations (7) and (8) represent the input-output relations for the 3DOF helicopter. Equations (9) represent the *internal dynamics* [30] or *driven dynamics* [20] of the 3DOF helicopter aircraft. The internal dynamics for the 3DOF helicopter system is then described in terms of the roll angle ϕ and its derivative $\dot{\phi}$ or, equivalently, η_1 and η_2.

The zero dynamics of the 3DOF helicopter can be obtained by constraining all the errors in Eqs. (9) to zero and assuming that $\ddot{\psi}_c = \ddot{\theta}_c = 0$,

$$\ddot{\eta}_1 = \frac{\sin\eta_1}{\epsilon} \tag{10}$$

or, equivalently, in terms of the original coordinates

$$\ddot{\phi} = \frac{\sin\phi}{\epsilon}$$

If $\epsilon > 0$, the zero dynamics [Eq. (10)] exhibits an unstable (saddle) equilibrium at the origin $\phi = \dot{\phi} = 0$. If $\epsilon < 0$, the eigenvalues of the zero dynamics are on the jw axis. The case when $\epsilon = 0$ is the subject of the next section because, in this very special case, the 3DOF helicopter model is feedback linearizable using dynamic extension and hence no zero dynamics [5]. Nonlinear systems, such as the 3DOF helicopter [Eqs. (7–9)], with zero dynamics [Eq. (10)] that are not asymptotically stable are called nonminimum phase [30].

Derivation of Minimum Phase Model

Let us define the following coordinate transformation:

$$\bar{\psi} = \psi - \epsilon\sin\phi \tag{11}$$

$$\bar{\theta} = \theta + \epsilon\cos\phi \tag{12}$$

$$\bar{u}_s = u_s - \epsilon\dot{\phi}^2 \tag{13}$$

Substituting the preceding in Eqs. (1–3), we obtain

$$\ddot{\bar{\theta}} = \bar{u}_s\cos\phi - \Omega^2 \tag{14}$$

$$\ddot{\bar{\psi}} = -\bar{u}_s\sin\phi \tag{15}$$

$$\ddot{\phi} = u_d \tag{16}$$

Remark 1: The representation of Eqs. (14–16) has the same structure as that of Eqs. (1–3) with $\epsilon = 0$. An interesting point is that the new output $\bar{\psi}$ and $\bar{\theta}$ is a flat output for the system [Eqs. (1–3)]. With such new outputs defined by Eqs. (11) and (12), it can be shown that the representation of Eqs. (14–16) is minimum phase and it can be fully linearized by dynamic extension [6].

Two Timescale Feedback Linearization

In this section, we present a new approach to design a tracking controller for the 3DOF helicopter model defined by Eqs. (14–16) such that the outputs defined by Eqs. (11) and (12) track a desired output defined by $\bar{\psi}_c(t)$ and $\bar{\theta}_c(t)$ asymptotically. In this approach, we separate the system into fast and slow subsystems using two timescale feedback control. First, a high-gain feedback linearizing controller is designed for Eq. (14) such that the tracking error $\bar{\theta} - \bar{\theta}_c(t)$ is forced to zero relatively fast. Second, a feedback linearizing controller is designed for Eq. (15) to derive the tracking error $\bar{\psi} - \bar{\psi}_c(t)$ to zero asymptotically. The second controller is designed under the assumption that the first output has converged to its desired value relatively fast. This approach has the advantage of achieving a lower order controller compared to the dynamic extension approach described in [6]. In fact, the dynamic extension approach will result in an eight-dimensional system compared to maintaining the system dimension in this approach to six states.

The feedback linearizing control for Eq. (14) is given by

$$\bar{u}_s = \frac{1}{\cos\phi}[v_1 + \Omega^2] \tag{17}$$

where v_1 is a new control input given by

$$v_1 = \ddot{\bar{\theta}}_c - \frac{k_2}{\varepsilon_1}(\dot{\bar{\theta}} - \dot{\bar{\theta}}_c) - \frac{k_1}{\varepsilon_1^2}(\bar{\theta} - \bar{\theta}_c) \tag{18}$$

The constants k_1 and k_2 are positive, and $\varepsilon_1 > 0$ is a timescale constant. Define the following error coordinates:

$$\bar{e}_1 = \frac{\bar{\theta} - \bar{\theta}_c}{\varepsilon_1^2} \tag{19}$$

$$\bar{e}_2 = \frac{\dot{\bar{\theta}} - \dot{\bar{\theta}}_c}{\varepsilon_1} \tag{20}$$

The closed-loop system using the preceding controller [Eqs. (17) and (18)] can be written as a two timescale system:

$$\varepsilon_1 \dot{\bar{e}}_1 = \bar{e}_2 \tag{21}$$

$$\varepsilon_1 \dot{\bar{e}}_2 = -k_2\bar{e}_2 - k_1\bar{e}_1 \tag{22}$$

$$\ddot{\bar{\psi}} = -[\ddot{\bar{\theta}}_c + \Omega^2 - k_2\bar{e}_2 - k_1\bar{e}_1]\tan\phi \tag{23}$$

$$\ddot{\phi} = u_d \tag{24}$$

where the first two equations represent the fast subsystem and, by setting $\varepsilon_1 = 0$, results in $\bar{e}_1 = \bar{e}_2 = 0$. Hence, the slow subsystem is given by

$$\ddot{\bar{\psi}} = -[\ddot{\bar{\theta}}_c + \Omega^2]\tan\phi \tag{25}$$

$$\ddot{\phi} = u_d \tag{26}$$

Remark 2: The slow subsystem (25) and (26) is of four-dimensional state space. An interesting point is that the system's relative degree with respect to the output $\bar{\psi}$ is four. Hence, the slow subsystem is fully linearizable without the need for additional integrators as in the case of dynamics extension.

Differentiating Eq. (25) twice until the input u_d appears

$$\dddot{\bar{\psi}} = -\dddot{\bar{\theta}}_c \tan\phi - [\ddot{\bar{\theta}}_c + \Omega^2]\frac{\dot{\phi}}{\cos^2\phi} \tag{27}$$

$$\ddddot{\bar{\psi}} = -\ddddot{\bar{\theta}}_c \tan\phi - 2\dddot{\bar{\theta}}_c \frac{\dot{\phi}}{\cos^2\phi} + \frac{\ddot{\bar{\theta}}_c + \Omega^2}{\cos^2\phi}[2\dot{\phi}^2\tan\phi - u_d] \tag{28}$$

Equation (28) is linearized using the control law

$$u_d = 2\dot{\phi}^2\tan\phi - \frac{\cos^2\phi}{\ddot{\bar{\theta}}_c + \Omega^2}\left[v_2 + \ddddot{\bar{\theta}}_c \tan\phi + 2\dddot{\bar{\theta}}_c \frac{\dot{\phi}}{\cos^2\phi}\right] \tag{29}$$

where v_2 is a new control law given by

$$v_2 = \ddddot{\bar{\psi}}_c - k_6(\dddot{\bar{\psi}} - \dddot{\bar{\psi}}_c) - k_5(\ddot{\bar{\psi}} - \ddot{\bar{\psi}}_c) - k_4(\dot{\bar{\psi}} - \dot{\bar{\psi}}_c) - k_3(\bar{\psi} - \bar{\psi}_c) \tag{30}$$

The feedback gains k_3, k_4, k_5, and k_6 are chosen such that the following polynomial

$$S^4 + k_6S^3 + k_5S^2 + k_4S + k_3 = 0 \tag{31}$$

is Hurwitz.

Let us define the following error coordinate for the slow subsystem (25) and (26) as

$$\bar{e}_3 = \bar{\psi} - \bar{\psi}_c \tag{32}$$

$$\bar{e}_4 = \dot{\bar{\psi}} - \dot{\bar{\psi}}_c \tag{33}$$

$$\bar{e}_5 = \ddot{\bar{\psi}} - \ddot{\bar{\psi}}_c \tag{34}$$

$$\bar{e}_6 = \dddot{\bar{\psi}} - \dddot{\bar{\psi}}_c \tag{35}$$

Using the control law (29) and (30) and the error coordinates (32) and (35), the closed-loop for the slow subsystem (25) and (26) can be written as

$$\dot{\bar{e}}_3 = \bar{e}_4 \tag{36}$$

$$\dot{\bar{e}}_4 = \bar{e}_5 \tag{37}$$

$$\dot{\bar{e}}_5 = \bar{e}_6 \tag{38}$$

$$\dot{\bar{e}}_6 = -k_6\bar{e}_6 - k_5\bar{e}_5 - k_4\bar{e}_4 - k_3\bar{e}_3 \tag{39}$$

Lemma 1: The slow subsystem (25) and (26) along with the nonlinear control law (29) and (30) and the error coordinates (32–35) is equivalent to the closed-loop system described by Eqs. (36–39).

Proof: The complete set of state and control transformation used to obtain the equivalent system (36–39) are as follows:

$$\bar{e}_3 = \bar{\psi} - \bar{\psi}_c \tag{40}$$

$$\bar{e}_4 = \dot{\bar{\psi}} - \dot{\bar{\psi}}_c \tag{41}$$

$$\bar{e}_5 = \ddot{\bar{\psi}} - \ddot{\bar{\psi}}_c = -[\ddot{\bar{\theta}}_c + \Omega^2]\tan\phi - \ddot{\bar{\psi}}_c \tag{42}$$

$$\bar{e}_6 = \dddot{\bar{\psi}} - \dddot{\bar{\psi}}_c = -\dddot{\bar{\theta}}_c \tan\phi - [\ddot{\bar{\theta}}_c + \Omega^2]\frac{\dot{\phi}}{\cos^2\phi} - \dddot{\bar{\psi}}_c \tag{43}$$

$$u_d = 2\dot{\phi}^2 \tan\phi - \frac{\cos^2\phi}{\ddot{\bar{\theta}}_c + \Omega^2}\left[v_2 + \ddddot{\bar{\theta}}_c \tan\phi + 2\dddot{\bar{\theta}}_c \frac{\dot{\phi}}{\cos^2\phi}\right] \quad (44)$$

$$v_2 = \ddddot{\bar{\psi}}_c - k_6\bar{e}_6 - k_5\bar{e}_5 - k_4\bar{e}_4 - k_3\bar{e}_3 \quad (45)$$

The state inverse transformation is defined as

$$\bar{\psi} = \bar{e}_3 + \bar{\psi}_c \quad (46)$$

$$\dot{\bar{\psi}} = \bar{e}_4 + \dot{\bar{\psi}}_c \quad (47)$$

$$\phi = -\tan^{-1}\frac{\bar{e}_5 + \ddot{\bar{\psi}}_c}{\ddot{\bar{\theta}}_c + \Omega^2} \quad (48)$$

$$\dot{\phi} = \frac{\dddot{\bar{\theta}}_c[\bar{e}_5 + \ddot{\bar{\psi}}_c] - [\ddot{\bar{\theta}}_c + \Omega^2][\bar{e}_6 + \dddot{\bar{\psi}}_c]}{[\ddot{\bar{\theta}}_c + \Omega^2]^2}\cos^2\phi \quad (49)$$

which is well defined. Thus, the closed-loop system (36–39) is equivalent to the slow subsystem (25) and (26).

Remark 3: Consider Eq. (42) in steady state (i.e., $\bar{e}_5 \longrightarrow 0$), the bank angle ϕ behaves as

$$\phi_{ss} = \tan^{-1}\frac{-\ddot{\bar{\psi}}_c}{\ddot{\bar{\theta}}_c + \Omega^2}$$

Thus, for any smooth commands $\psi_c(t)$ and $\theta_c(t)$, the flat output commands $\bar{\psi}_c(t)$ and $\bar{\theta}_c(t)$ can be calculated using Eqs. (11) and (12) as

$$\bar{\psi}_c = \psi_c - \epsilon \sin\phi_{ss}$$

$$\bar{\theta}_c = \theta_c + \epsilon \cos\phi_{ss}$$

For commands satisfying $\ddot{\bar{\psi}}_c = 0$, the transformation is straightforward as

$$\bar{\psi}_c = \psi_c$$

$$\bar{\theta}_c = \theta_c + \epsilon$$

Main Result

Lemma 2: The tracking problem for the 3DOF helicopter system

$$\ddot{\theta} = u_s \cos\phi + \epsilon u_d \sin\phi - \Omega^2 \quad (50)$$

$$\ddot{\psi} = -u_s \sin\phi + \epsilon u_d \cos\phi \quad (51)$$

$$\ddot{\phi} = u_d \quad (52)$$

with outputs

$$\bar{\psi} = \psi - \epsilon \sin\phi \quad (53)$$

$$\bar{\theta} = \theta + \epsilon \cos\phi \quad (54)$$

is asymptotically stable for any sufficiently smooth commands $\bar{\psi}_c(t)$ and $\bar{\theta}_c(t)$ with the use of nonlinear two timescale feedback control defined by

$$u_s = \bar{u}_s + \epsilon\dot{\phi}^2 \quad (55)$$

$$\bar{u}_s = \frac{1}{\cos\phi}[v_1 + \Omega^2] \quad (56)$$

$$v_1 = \ddot{\bar{\theta}}_c - k_2\bar{e}_2 - k_1\bar{e}_1 \quad (57)$$

$$u_d = 2\dot{\phi}^2 \tan\phi - \frac{\cos^2\phi}{\ddot{\bar{\theta}}_c + \Omega^2}\left[v_2 + \ddddot{\bar{\theta}}_c \tan\phi + 2\dddot{\bar{\theta}}_c \frac{\dot{\phi}}{\cos^2\phi}\right] \quad (58)$$

$$v_2 = \ddddot{\bar{\psi}}_c - k_6\bar{e}_6 - k_5\bar{e}_5 - k_4\bar{e}_4 - k_3\bar{e}_3 \quad (59)$$

where

$$\bar{e}_1 = \frac{\bar{\theta} - \bar{\theta}_c}{\varepsilon_1^2} \quad (60)$$

$$\bar{e}_2 = \frac{\dot{\bar{\theta}} - \dot{\bar{\theta}}_c}{\varepsilon_1} \quad (61)$$

$$\bar{e}_3 = \bar{\psi} - \bar{\psi}_c \quad (62)$$

$$\bar{e}_4 = \dot{\bar{\psi}} - \dot{\bar{\psi}}_c \quad (63)$$

$$\bar{e}_5 = \ddot{\bar{\psi}} - \ddot{\bar{\psi}}_c = -[\ddot{\bar{\theta}}_c + \Omega^2]\tan\phi - \ddot{\bar{\psi}}_c \quad (64)$$

$$\bar{e}_6 = \dddot{\bar{\psi}} - \dddot{\bar{\psi}}_c = -\dddot{\bar{\theta}}_c \tan\phi - [\ddot{\bar{\theta}}_c + \Omega^2]\frac{\dot{\phi}}{\cos^2\phi} - \dddot{\bar{\psi}}_c \quad (65)$$

Proof: The resulting closed-loop system using the preceding state and control transformation (55–65) can be written as a two timescale system

$$\varepsilon_1\dot{\bar{e}}_1 = \bar{e}_2 \quad (66)$$

$$\varepsilon_1\dot{\bar{e}}_2 = -k_2\bar{e}_2 - k_1\bar{e}_1 \quad (67)$$

$$\ddot{\bar{\psi}} = -[\ddot{\bar{\theta}}_c + \Omega^2 - k_2\bar{e}_2 - k_1\bar{e}_1]\tan\phi \quad (68)$$

$$\ddot{\phi} = u_d \quad (69)$$

where the first two equations represent the fast subsystem and, by setting $\varepsilon_1 = 0$, results in $\bar{e}_1 = \bar{e}_2 = 0$. Hence, the slow subsystem is given by

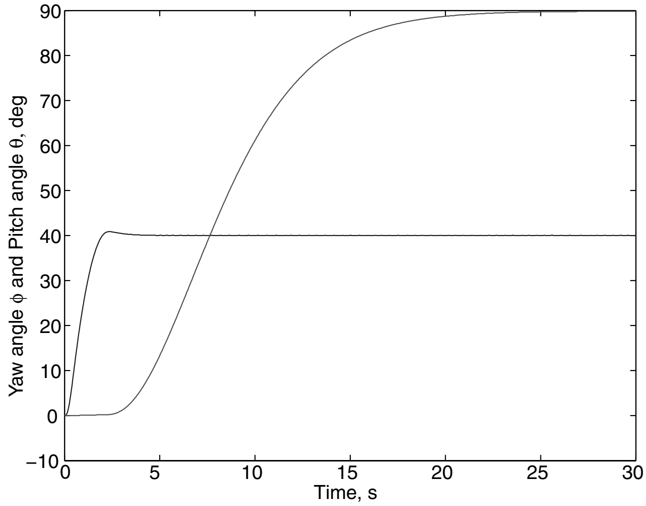

Fig. 2 Aircraft response to step commands in pitch ($\bar{\theta}_c = 40\,\mathrm{deg}$) and yaw ($\bar{\psi}_c = 90\,\mathrm{deg}$) angles.

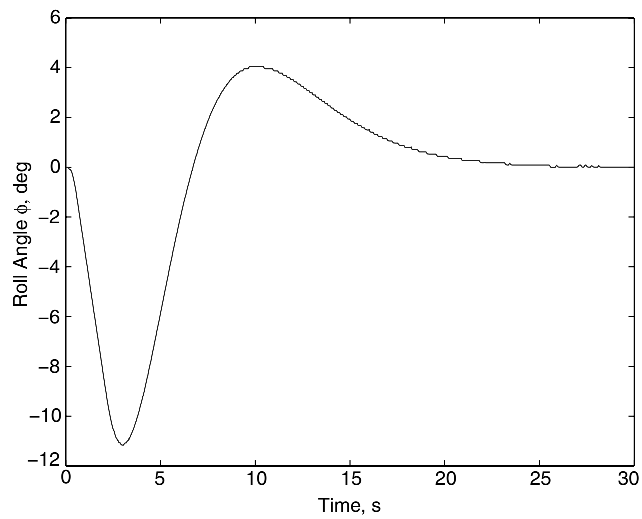

Fig. 3 Roll angle response to step commands in pitch ($\bar{\theta}_c = 40\,\mathrm{deg}$) and yaw ($\bar{\psi}_c = 90\,\mathrm{deg}$) angles.

$$\ddot{\bar{\psi}} = -[\ddot{\bar{\theta}}_c + \Omega^2]\tan\phi \tag{70}$$

$$\ddot{\phi} = u_d \tag{71}$$

The origin of the boundary-layer system [fast subsystem, Eqs. (66) and (67)] is exponentially stable by construction. According to Lemma 1, the slow subsystem is equivalent to the closed-loop system described by Eqs. (36–39), which can be made exponentially stable by appropriate choice of the gains k_3, k_4, k_5, and k_6. Consequently, the result follows according to the singular perturbation results in [31] (p. 380).

Remark 4: Usually, for flight control problems, the fast subsystem is the roll dynamics and the slow subsystem is the pitch–yaw dynamics. In this work, the pitch dynamics is the fast subsystem for two reasons: First, to decouple it from the roll–yaw subsystem and hence simplify the stability analysis. Second, the 3DOF helicopter model resembles a vertical takeoff and landing aircraft and thus fast takeoff (clearing from ground to hover position) is an advantage which can be achieved by having fast pitch dynamics. Furthermore, the concept of the two timescale approach simply means that there is a relative speed of convergence between the fast and the slow subsystems, and not necessary the use of infinite (or large) gain for the fast subsystem.

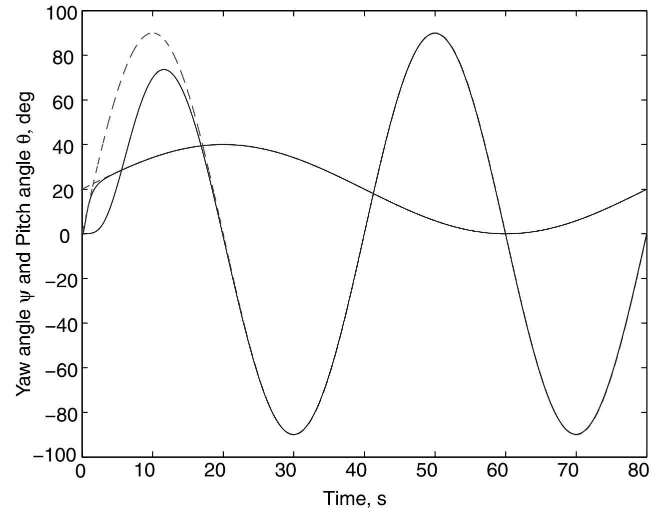

Fig. 4 Aircraft response to commands in pitch ($\bar{\theta}_c = 20[1 + \sin(\pi/40)t]$) and yaw ($\bar{\psi}_c = 90[\sin(\pi/20)t]$) angles.

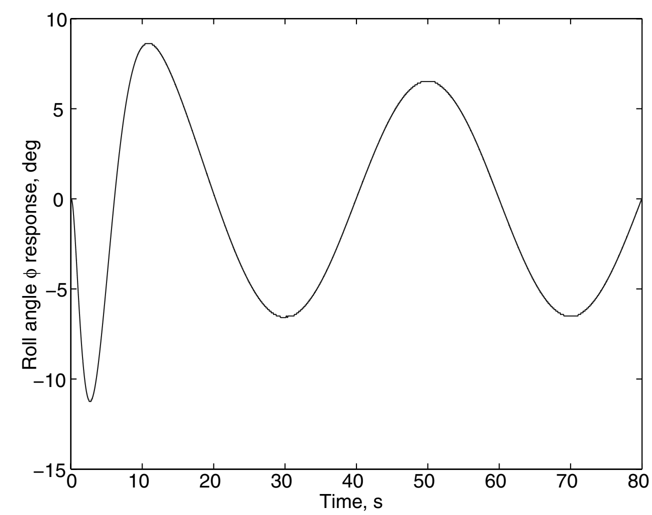

Fig. 5 Roll angle response to commands in pitch ($\bar{\theta}_c = 20[1 + \sin(\pi/40)t]$) and yaw ($\bar{\psi}_c = 90[\sin(\pi/20)t]$) angles.

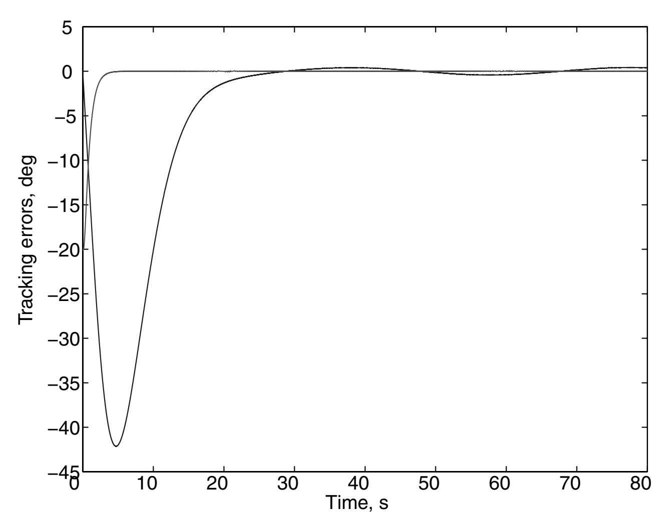

Fig. 6 Tracking errors to commands in pitch ($\bar{\theta}_c = 20[1 + \sin(\pi/40)t]$) and yaw ($\bar{\psi}_c = 90[\sin(\pi/20)t]$) angles.

Numerical Results

For the purpose of numerical illustration, we provide simulations for the 3DOF helicopter model described by Eqs. (14–16). Figures 2 and 3 show the aircraft responses to step commands of 40 and 90 deg in pitch and yaw angles, respectively. It is clear that outputs converge to the desired values and all internal states and control signals are bounded. Figures 4 and 5, show the effectiveness of the proposed control scheme in tracking time-varying signals for pitch and yaw angles. Figure 6 shows that all tracking errors converge to zero.

Conclusions

In this paper, we have studied the output tracking problem for a 3DOF nonlinear nonminimum phase flight vehicle model. The results indicate that a semiglobal output tracking problem for such a vehicle can be solved, after state and control transformation, by separating the aircraft dynamics into fast and slow subsystems using two timescale feedback control. First, a high-gain feedback linearizing controller is designed for the fast subsystem such that the tracking error is forced to zero relatively fast. Second, a feedback linearizing controller is designed for the slow subsystem to derive the tracking error to zero asymptotically. This approach has the advantage of achieving a lower order controller compared to the dynamic extension approach. Simulation results were provided to demonstrate the effectiveness of such a control design approach.

References

[1] Spong, M., Khorasani, K., and Kokotovic, P., "An Integrated Manifold Approach to the Feedback Control of Flexible Joint Robots," *IEEE Journal of Robotics and Automation*, Vol. 38, No. 4, 1987, pp. 291–300.

[2] Zhao, H., and Chen, D., "Tip Trajectory Tracking for Multilink Flexible Manipulators Using Stable Inversion," *Journal of Guidance, Control, and Dynamics*, Vol. 21, No. 2, 1998, pp. 314–320.
doi:10.2514/2.4237

[3] Laudval, T., and Fossen, T. I., "Nonlinear Non-Minimum Phase Rudder-Roll Damping System for Ships Using Sliding Mode Control," *Proceedings of the European Control Conference*, International Federation of Automatic Control, 1997.

[4] Benvenuti, L., Di Benedetto, M. D., and Grizzle, J. W., "Approximate Output Tracking for Nonlinear Non-Minimum Phase System with an Application to Flight Control," *International Journal of Robust and Nonlinear Control*, Vol. 4, No. 3, 1994, pp. 397–414.
doi:10.1002/rnc.4590040307

[5] Hauser, J., Sastry, S., and Meyer, G., "Nonlinear Controller Design For Slightly Non-Minimum Phase Systems: Application To V/STOL Aircraft," *Automatica*, Vol. 28, No. 4, 1992, pp. 665–679.
doi:10.1016/0005-1098(92)90029-F

[6] Martin, P., Devasia, S., and Paden, B., "A Different Look at Output Tracking: Control of a VTOL Aircraft," *Proceedings of 33rd IEEE Conference on Decision and Control*, Inst. of Electrical and Electronics Engineers, New York, 1994, pp. 2376–2381.

[7] Reiner, J., Balas, G. L., and Garrard, W. L., "Flight Control Design Using Robust Dynamic Inversion and Time Scale Separation," *Automatica*, Vol. 32, No. 11, 1996, 1493–1504.
doi:10.1016/S0005-1098(96)00101-X

[8] Tomlin, C. J., and Sastry, S. S., "Bounded Tracking for Nonminimum Phase Nonlinear System with Fast Zero Dynamics," *International Journal of Control*, Vol. 68, No. 4, 1997, pp. 819–857.
doi:10.1080/002071797223352

[9] Devaud, E., Font, S., and Siguerdidjane, H., "Some Control Strategies of High Angle of Attack Missile Autopilot," *14th IFAC Symposium on Automatic Control*, Elsevier, New York, 1998.

[10] Harcaut, J. P., "Nonlinear Control of a Missile Through a Geometric Approach," *Lecture Notes in Control and Information Sciences*, Vol. 144, Edited by A. Bensoussan, and J. L. Lions, Springer–Verlag, New York, 1990, pp. 27–36.

[11] Menon, P. K., and Yousefpor, M. X., "Design of Nonlinear Autopilots for High Angle of Attack Missiles," *AIAA Guidance Navigation and Control Conference*, AIAA Paper 96-3913, 1996.

[12] Nichols, R. A., Reichert, R. T., and Rugh, W. J., "Gain Scheduling for *h*-Infinity Controllers: A Flight Control Example," *IEEE Transactions on Control Systems Technology*, Vol. 1, No. 2, 1993, pp. 69–79.
doi:10.1109/87.238400

[13] Wu, F., Packard, A., and Balas, G., "Lpv control design for pitch-axis missile autopilots," *Proceedings of the 1999 34th IEEE Conference on Decision and Control*, Inst. of Electrical and Electronics Engineers, New York, 1995, pp. 188–193.

[14] Slotine, J. E., and Li, W., *Applied Nonlinear Control*, Prentice–Hall, Upper Saddle River, NJ, 1991.

[15] Shtessel, Y., "Nonlinear Nonminimum Phase Output Tracking via Dynamic Sliding Manifolds," *Journal of the Franklin Institute*, Vol. 335B, No. 5, 1998, pp. 841–850.
doi:10.1016/S0016-0032(97)00014-8

[16] Shtessel, Y., and Tournes, C., "Nonminimum Phase Output Tracking in Dynamic Sliding Manifolds with Application to Aircraft Control," *Proceedings of the 35th IEEE Conference on Decision and Control*, Inst. of Electrical and Electronics Engineers, New York, 1996, pp. 2071–2076.

[17] Isidori, A., and Byrnes, C., "Output Regulation of Nonlinear Systems," *IEEE Transactions on Automatic Control*, Vol. 35, No. 2, 1990, pp. 131–140.
doi:10.1109/9.45168

[18] Francis, B. A., "The Linear Multivariable Regulator Problem," *SIAM Journal on Control and Optimization*, Vol. 15, No. 3, 1977, pp. 486–505.
doi:10.1137/0315033

[19] Devasia, S., Chen, D., and Paden, B., "Nonlinear Inversion-Based Output Tracking," *IEEE Transactions on Automatic Control*, Vol. 41, No. 7, 1996, pp. 930–942.
doi:10.1109/9.508898

[20] Meyer, G., Hunt, L. R., and Murray, R. M., "Nonlinear System Guidance in the Presence of Transmission Zero Dynamics," NASA TR 4661, 1995.

[21] Tomlin, C., Lygeros, J., Benvenuti, L., and Sastry, S., "Output Tracking for a Non-Minimum Phase Dynamic CTOL Aircraft Model," *Proceedings of the 34th IEEE Conference on Decision and Control*, Inst. of Electrical and Electronics Engineers, New York, 1995, pp. 1867–1872.

[22] DeSantis, R. M., "Modeling and Path-Tracking for a Load-Haul-Dump Mining Vehicle," *Journal of Dynamic Systems, Measurement, and Control*, Vol. 119, No. 1, 1997, pp. 40–47.
doi:10.1115/1.2801212

[23] Tarn, T., Xi, N., and Bejczy, A. K., "Path-Based Approach to Integrated Planning and Control for Robotic Systems," *Automatica*, Vol. 32, No. 12, 1996, pp. 1675–1687.
doi:10.1016/S0005-1098(96)80004-5

[24] Al-Hiddabi, S., and McClamroch, N. H., "Tracking and Maneuver Regulation Control for Nonlinear Non-Minimum Phase Systems: Application to Flight Control," *IEEE Transactions on Control Systems Technology*, Vol. 10, No. 6, 2002, pp. 780–792.
doi:10.1109/TCST.2002.804120

[25] Hauser, J., and Hindman, R., "Aggressive Flight Maneuvers," *Proceedings of the IEEE Conference on Decision and Control*, Inst. of Electrical and Electronics Engineers, New York, 1997, pp. 4186–4191.

[26] Kaminer, I., Pascoal, A., Hallberg, E., and Silvestre, C., "Trajectory Tracking for Autonomous Vehicles: An Integrated Approach to Guidance and Control," *Journal of Guidance, Control, and Dynamics*, Vol. 21, No. 1, 1998, pp. 29–38.
doi:10.2514/2.4229

[27] Hauser, J., and Hindman, R., "Maneuver Regulation from Trajectory Tracking: Feedback Linearizable Systems," *Nonlinear Control System Design 1995 from the 3rd IFAC Symposium* (Postprint), Vol. 1, Pergamon, Oxford, England, U.K., 1996, pp. 269–274.

[28] Hindman, R., and Hauser, J., "Stability of Maneuver Regulation Using Arbitrary Transverse Foliations," *American Control Conference*, American Automatic Control Council, Evanston, IL, 1999, pp. 4558–4562.

[29] Olfati-Saber, R., "Global Configuration Stabilization for the VTOL Aircraft with Strong Input Coupling," *IEEE Transactions on Automatic Control*, Vol. 47, No. 11, 2002, pp. 1949–1952.
doi:10.1109/TAC.2002.804457

[30] Isidori, A., *Nonlinear Control Systems: An Introduction*, Springer–Verlag, New York, 1985.

[31] Khalil, H. K., *Nonlinear systems*, Prentice–Hall, Upper Saddle River, NJ, 1996.